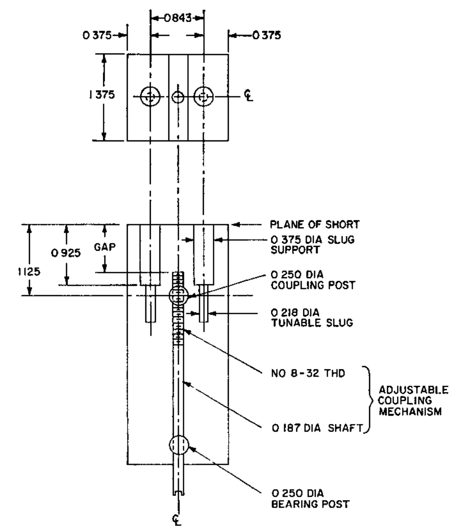

Fig. 1. Comb-line filter structure with modified interstage coupling mechanism.

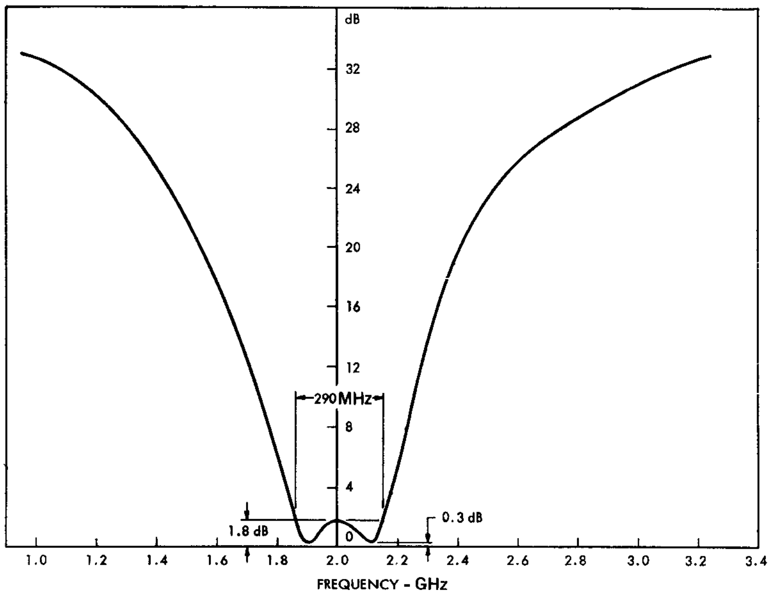

Fig. 2. Measured filter response (insertion loss versus frequency).

sured coupling bandwidth was 280 MHz. Upon further insertion of the coupling screw, reducing the gap to 0.730 inch, the measured coupling bandwidth was 353 MHz.

Both the coupling post and the coupling screw (as used herein) increased the net magnetic coupling and provided closer coupling between adjacent resonators. It is anticipated that both the coupling post and the coupling screw techniques can be used to obtain appreciable magnetic coupling between quarter-wave *uniform* comb-line resonators. Such uniform resonators are normally decoupled due to cancellation of magnetic and electric fields.[2]

The comb-line structure of Fig. 1, employing the 0.250 inch diameter coupling post, has been converted into a two-resonator bandpass filter by adding input/output couplings. Direct taps were implemented by using number 12 wire between the inner conductors of type UG-290A/U BNC fittings and tap points on the 0.375 inch diameter slug supports. With input/output tap points located 0.222 inch from the plane of the short, the measured filter response curve of Fig. 2 was obtained. Passband insertion loss comprised a 1.5 dB ripple plus 0.3 dB dissipation loss. The measured ripple bandwidth was about 290 MHz. The insertion loss on the high-frequency skirt drops down to 32 dB at 3500 MHz and 11 dB at 4000 MHz. This is due to the 1.375 inch dimension of the filter enclosure, which can propagate the TE_{10} rectangular waveguide mode at 4280 MHz.

Inductive posts have been used extensively as interstage couplings in waveguide bandpass filters. The brief experiments reported herein suggest the efficacy of using coupling posts in TEM filters employing comb-line resonators. Multiresonator bandpass filters can be fabricated quite economically using round-rod resonators parallel to the broad walls of nonpropagating rectangular waveguide tubing. Fixed resonator spacings could accommodate a range of filter bandwidths and/or response shapes by the use of coupling posts.

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High-Frequency Transistor Evaluation by Three-Port Scattering Parameters

Introduction

Semiconductor technology has developed sufficiently for transistors to retain useful properties up into the microwave region. One GHz is the approximate lower-frequency limit of this region, although a preferable concept might be to consider a transistor in microwave terms when the encapsulation design and associated jig or holder impedances have significant effect. One such effect is the reduction of measurement accuracy for admittance parameters, due to the increasing difficulty of specifying reference planes. Frequency characteristics of the jig and its interaction with the device must also be considered.

Transistors have so far been characterized as two-port networks whose admittance parameters are measured in two-port configurations. An alternative and more general method of three-terminal device characterization is by the three-port or three-terminal pair scattering parameters. The theory and technique for using scattering parameters are well known for multiport passive microwave junctions. The purpose of this correspondence is to apply the wave scattering concept to the evaluation of transistors.

Ideally, the three-terminal device is inserted at the center of the junction of three transmission lines. Reflection and transmission measurements made on each port then specify the device completely. An active device, in these circumstances, might exhibit circulation or gain, which could become appropriate quantities for characterization in the microwave region.

Because each port sees the characteristic waveguide impedance, power gain, circulation and dissipation in the matched arms of the junction can take place. Thus, algebraic expressions for the scattering parameters take on a more complicated form than the admittance parameters in terms of the transistor equivalent circuit elements. However, the practical consequences of measuring scatter-

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ing parameters are advantageous. Apart from the improvements which can be made in mounting the transistor and in specifying reference planes, the presence of matched ports should improve device stability during measurements. That is, oscillations and instabilities experienced with some two-port measurements may be eliminated, as the open- or short-circuit conditions (for admittance parameters) are no longer required.

Evaluation of Scattering Elements for a Simple Transistor Model

The scattering matrix contains elements denoting the amplitude and phases of waves scattered by the junction. Whereas network admittance parameters are defined at the accessible nodes, scattering parameters are defined for a particular reference plane. Hence the three-terminal network $[Y]$, defined in Fig. 1, is shown connected as a scattering network in Fig. 2, and the well-known matrix transformation from normalized admittance parameters $[Y]$ to scattering parameters $[S]$ is

$$[S] = ([1] - [Y])([1] + [Y])^{-1}. \quad (1)$$

In order to examine the behavior of S-parameters for transistors, the ideal model of Fig. 3 is considered. g_e is the conductance of the forward biased emitter-base junction and $g_m V$ is the current generator representing the collector action. The corresponding admittance matrix is, therefore,

$$[Y] = \begin{bmatrix} g_e & -g_e & 0 \\ g_m - g_e & g_e - g_m & 0 \\ -g_m & g_m & 0 \end{bmatrix}. \quad (2)$$

Ths substitution for obtaining $[S]$ requires normalized values of $[Y]$. For convenience, let $[Y]$ be normalized to g_e since the low-frequency simplification for common base current gain is

$$\frac{g_m}{g_e} = \alpha. \quad (3)$$

Thus, in terms of α, the S-matrix is

$$[S] = \begin{bmatrix} \dfrac{1-\alpha}{3-\alpha} & \dfrac{2}{3-\alpha} & 0 \\ 2\left(\dfrac{1-\alpha}{3-\alpha}\right) & \dfrac{1+\alpha}{3-\alpha} & 0 \\ \dfrac{2\alpha}{3-\alpha} & \dfrac{-2\alpha}{3-\alpha} & 1 \end{bmatrix}. \quad (4)$$

A physical interpretation of $[S]$ in (3) is obtained by putting α equal to unity, so that

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}. \quad (5)$$

Thus, port 1 is matched, ports 2 and 3 are totally reflecting. Zero transmission is produced from ports 1 to 2, 3 to 1, and 3 to 2. Transmission is produced from ports 1 to 3, 2 to 1, and 2 to 3. The scattering elements display an immediate correspondence to the expected performance of the network as drawn in Fig. 4, except S_{12} and S_{32}. However, input to port 2 corresponds to an input into the base of the transistor. Therefore it might be expected that a small input (which becomes zero for $\alpha=1$, corresponding to total reflection in the port) is associated with a large current flow in the emitter and collector branches. Thus a form of active circulation is produced by insertion of the transistor equivalent network into the three-port junction.

Property of the $[S]$-Matrix Derived from an Indefinite $[Y]$-Matrix

Equations (4) and (5) are the results of the transformation from normalized $[Y]$ to $[S]$ parameters for the special case of the transistor model in Fig. 1. It is readily seen that the rows and columns of $[S]$ add up to unity. The result

$$\sum_{j=1}^{3} S_{jk} = \sum_{k=1}^{3} S_{jk} = 1 \quad (6)$$

is found to be a general property of the $[S]$-matrix by summing appropriate elements from (1) and using the relation

$$\sum_{j=1}^{3} Y_{jk} = \sum_{k=1}^{3} Y_{jk} = 0 \quad (7)$$

for the indefinite matrix $[Y]$. In other words, the application of Kirchhoff's laws to $[Y]$ produces this characteristic of $[S]$. To obtain (6) it is not required that $Y_{jk} = Y_{kj}$ and therefore the result is applicable to active networks.

The practical consequences of this result are valuable. Although measurements of the nine scattering elements may be used to obtain $[Y]$, in principle scattering measurements made on two ports only are required, while the third port is terminated by the characteristic line admittance. Therefore only four elements of the S-matrix need to be measured, and, for example, reflection elements involving slotted line measurements may be discarded. Thus a measurement technique to combine accuracy and/or simplicity can be chosen with some flexibility.

Comparison of $[S]$ and $[Y]$ Measurement Techniques

Valid comparison requires compatibility of reference planes and access line impedances, and it is observed experimentally that apparently small changes in these conditions can cause significant variations in the directly measured Y-parameters of active networks.

In the two-port Y-parameter technique access to and termination of the device terminals is attempted as shown in Fig. 5(a). For the conventional transistor encapsulations, terminals 1, 2, and 3 refer to the lead at the header exit point. One terminal, in this case shown as terminal 3, is mechanically grounded in the device jig. As the frequency is increased, the electrical length x becomes an increasingly significant fraction of a wavelength. Also, the geometry of the mechanical ground is such that Y_0' is not specified.

In the S-method, the Y-parameters which are reconstituted from S-measurements obtained from the configuration of Fig. 5(b), are automatically referred to the device terminals. Changes in phase and amplitude of the transmitted waves are measured between the conditions when terminals 1, 2, and 3 are connected with lines of impedance Y_0 and with the transistor, T. Reflection coefficients measured on each port are referred to the open

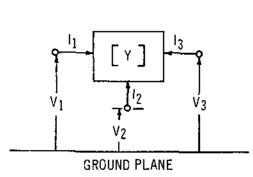

Fig. 1. Definition of currents and voltages for a 3-pole network $[Y]$.

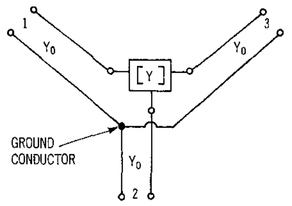

Fig. 2. Series connected scattering network for $[Y]$.

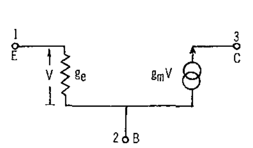

Fig. 3. Transistor model.

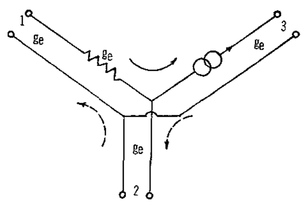

Fig. 4. Ideal transistor model in microwave junction.

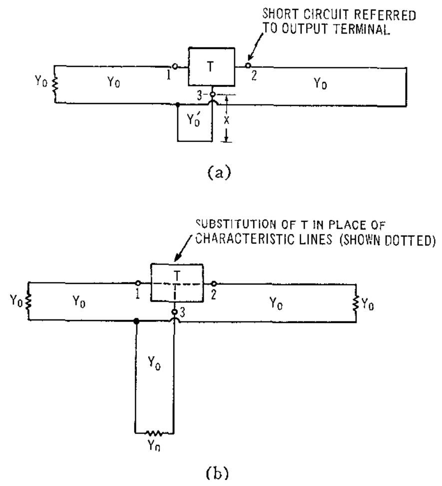

Fig. 5. Diagrams showing the two measurement conditions for T. (a) Two-port Y-method: measurement of current and voltages at terminals 1 and 2. (b) Three-port S-method: comparison of wave amplitude and phase in each line.

circuits which exist when the jig is empty. However, the property of the S-matrix shown in (6) refers the S-elements to the center of a three-port junction containing the negligibly small lumped element network. Therefore if scattering measurements are made on only two ports and the three-port S-matrix constructed, errors due to the physical size of the device in relation to the wavelength will be introduced. For the active semiconductor region itself, such error is negligible for frequencies at which transistors are likely to operate in the near future, but the encapsulations are sufficiently large to produce errors. However, if future encapsulations are designed to provide access lines of characteristic admittance up to the semiconductor chip itself, then this source of error will be insignificant.

Appendix[1]

Proof of Scattering Matrix Result

The foregoing correspondence states that "it is readily seen that the rows and columns of $[S]$ (the scattering matrix in Anderson's notation) add to unity." Since the general proof of this does not appear in any of the texts and works consulted either on circuit theory or matrix algebra it is presented as follows.

If two matrices $[A]$ and $[B]$ are such that their rows and/or columns sum to a, b, respectively, then it is evident that the rows and/or columns of the matrix $[A+B]$ will sum to $a+b$ and less obvious (but true) that their product sums to the product ab. Let us check the product property for, say, the row sum. The row sum is as follows:

$$\sum_k [AB]_{ik} = \sum_k \sum_j A_{ij}B_{jk}$$
$$= \sum_j A_{ij} \sum_k B_{jk} = \sum A_{ij}b = ab. \quad (8)$$

The column sum property is proved in the same manner.

Now the unit matrix $[1]$ obviously sums to one for both rows and columns. Using this fact, the product property and argument similar to (8) one can prove a useful property of the inverse matrix $[A^{-1}]$, by considering $[A][A^{-1}]=[1]$ for column sums and $[A^{-1}][A]=[1]$ for row sums. The property is this: the inverse of a matrix whose rows and/or columns sum to a is a matrix whose rows and/or columns sum to a^{-1}. Now the scattering matrix $[S]$ is given by

$$[S] = ([1] - [Y])([1] + [Y])^{-1} \quad (9)$$

and we are told that the normalized admittance matrix $[Y]$ has, by Kirchhoff's laws, rows and columns which sum to zero. The row and column sum of the scattering matrix $[S]$ is immediately $[(1-0)(1+0)^{-1}=1]$ one.

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[1] The Appendix to this correspondence was written by T. W. Johnston, RCA Victor Plasma and Space Physics Research Laboratory, Montreal, Canada.

Transmission-Line Treatment of Waveguides Filled with a Moving Medium

To the author's knowledge, a theoretical study of guided waves in a moving isotropic medium was first investigated by Collier and Tai.[1] They derived the electromagnetic fields within a source-free region of a circular or rectangular waveguide by the method of vector potentials.

This correspondence discusses the problem of determining the vector fields produced by arbitrary electric and magnetic impressed currents in a uniform waveguide of arbitrary cross section filled with a dielectric medium, which moves down the waveguide with a constant velocity, by representing the fields in terms of a suitable set of vector mode functions.

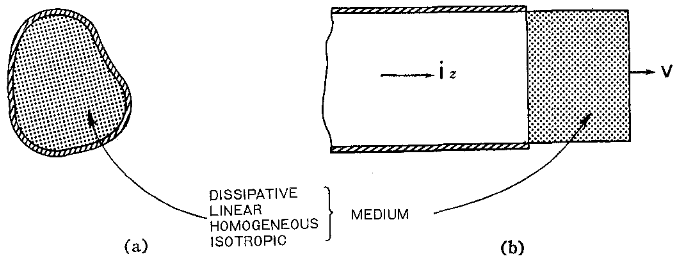

Fig. 1. Uniform waveguide of arbitrary cross section filled with a moving medium. (a) Cross sectional view. (b) Longitudinal view.

Let us assume that a homogeneous, isotropic, and lossy medium is moving with constant velocity $v = vi_z$, past an observer at rest with respect to the waveguide (see Fig. 1). As long as the velocity of the medium is much smaller than the velocity of light, the electromagnetic fields inside a waveguide, measured in the rest frame of the observer, can be determined by the following Maxwell-Minkowski equations:[2]

$$(\nabla - j\omega\mathbf{\Lambda}) \times E = -j\omega\mu H - J^*$$
$$(\nabla - j\omega\mathbf{\Lambda} - \mu\sigma v) \times H = j\omega\hat{\epsilon}E + J \quad (1)$$

where

$\mathbf{\Lambda} = (\epsilon\mu - \epsilon_0\mu_0)v = \Lambda i_z,\ \Lambda = (\epsilon\mu - \epsilon_0\mu_0)v$
$\hat{\epsilon} = \epsilon(1 - j\sigma/\omega\epsilon)$
ϵ_0, μ_0 = permittivity and permeability of free space
ϵ, μ, σ = permittivity, permeability, and conductivity of the medium at rest
J, J^* = impressed electric and magnetic current densities

and the time variation of the fields has been assumed to be $e^{+j\omega t}$.

To obtain the solution of the foregoing inhomogeneous field equations, we first transform these equations into more familiar forms. This can be done by letting

$$E = E_0 e^{+j\omega\Lambda z}, \quad H = H_0 e^{+j\omega\Lambda z}$$
$$J = J_0 e^{+j\omega\Lambda z}, \quad J^* = J_0^* e^{+j\omega\Lambda z}. \quad (2)$$

Substituting (2) into (1), we obtain the following equations for E_0 and H_0:

$$\nabla \times E_0 = -j\omega\mu H_0 - J_0^*$$
$$(\nabla - \mu\sigma v) \times H_0 = j\omega\hat{\epsilon}E_0 + J_0. \quad (3)$$

Even if the medium involved is moving with constant velocity along the perfectly conducting guide walls, the electric and magnetic fields should satisfy the same boundary conditions as in the case of stationary media:[3]

$$n \times E = 0 \qquad n \times E_0 = 0$$
$$n \cdot H = 0 \quad \text{or} \quad n \cdot H_0 = 0 \quad (4)$$

where n denotes a unit vector normal to the guide walls. The electric and magnetic fields E_0 and H_0 which satisfy the inhomogeneous field equations (3) and subject to the boundary conditions (4), are then expressed in the following forms in terms of the vector mode functions:

$$E_{0t} = \sum_i [V_i^e(z)M_i^e + V_i^m(z)M_i^m]$$
$$H_{0t} = \sum_i [-I_i^e(z)N_i^e + I_i^m(z)N_i^m]$$
$$E_{0z} = \sum_i [(k_i^e)^2 V_{zi}(z)M_{zi}]$$
$$H_{0z} = \sum_i [(k_i^m)^2 I_{zi}(z)N_{zi}] \quad (5)$$

where V_i^e, V_i^m, V_{zi} and I_i^e, I_i^m, I_{zi} are mode voltages and currents, respectively. The superscripts e and m denote the electric (or TM) modes and the magnetic (or TE) modes, respectively. Also, the subscripts t and z are employed to designate the transverse field components and the longitudinal ones, respectively.

The vector mode functions M_i^e, N_i^e, M_{zi} and M_i^m, N_i^m, N_{zi} are characterized by the following equations:

$$M_i^e = \nabla_t\Phi_i^e, \qquad M_i^m = \nabla_t\Phi_i^m \times i_z$$
$$N_i^e = \nabla_t\Phi_i^e \times i_z, \qquad N_i^m = \nabla_t\Phi_i^m$$
$$M_{zi} = \Phi_i^e i_z, \qquad N_{zi} = \Phi_i^m i_z \quad (6)$$

where the functions Φ_i^e and Φ_i^m are derived from the scalar eigenvalue problems

$$\nabla_t^2\Phi_i^e + (k_i^e)^2\Phi_i^e = 0$$
$$\nabla_t^2\Phi_i^m + (k_i^m)^2\Phi_i^m = 0 \quad (7)$$

subject to

$$\Phi_i^e = 0$$
$$\partial\Phi_i^m/\partial n = 0 \quad (8)$$

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[1] J. R. Collier, and C. T. Tai, "Guided waves in moving media," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-13, pp. 441–445, July 1965.

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